# EXPERIMENTAL DETERMINATION OF THE COEFFICIENTS OF RESTITUTION OF PARTICLES IN THE FLOW OF A GAS SUSPENSION IN A COLLISION AGAINST THE SURFACE

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The conditions for performing measurements of the coefficients of restitution of particles colliding with a plate positioned at an angle with respect to the direction of flow of a gas suspension were studied. Experimental data were obtained for surfaces of different materials.

In solving a number of problems in the mechanics of heterogeneous media, for example, the problem of determining the resistance of a body in a flow of gas containing solid particles and the calculation of the motion of a gas suspension in complicated channels, it is necessary to describe the dynamics of collisions of particles with a surface. In the case of the interaction of a body with a high-velocity two-phase flow containing irregularly shaped solid particles, when residual deformations and fracture of the particle material and the body are observed, the modern theory of impact cannot describe the mechanics of the impact interaction of particles with a solid, so that experimentally determined coefficients of restitution of the particles are employed in order to solve practical problems.

For a number of reasons, the kinematic parameters of a particle recoiling from a surface are of a random character, so that the coefficient of restitution of the particle must be interpreted in a statistical sense. For practical purposes, the statistical-average coefficient of restitution is, evidently, of greatest value. In [1, 2] the statistical average coefficient of restitution is determined with the help of an analysis of the statistical stability of an empirical law governing the character of the impact interaction of a sample of particles with a surface. The method of investigation of coefficients of restitution which is described in [1, 2] is laborious and requires complicated calculations. In this work the statistical average coefficients of restitution for particles in the flow of a gaseous suspension are measured experimentally based on the total force exerted by the particles on the sample under study.

We shall study the interaction of solid particles in a two-phase flow with a plate of area $\sigma$, positioned at an angle $\alpha$ with respect to the velocity vector $U_\infty$ of the incident particles (Fig. 1). The distributions of the density of the discrete phase $\rho_W$ and the particle velocity $U_W$ at the surface of the plate before the collision are known. Erosion of the surface of the plate and fragmentation of the particles were neglected. Then, using the law of conservation of momentum and introducing the coefficient of restitution for the normal $a_n$ and tangential $a_\tau$ components of the particle velocity

$$a_n = \frac{|U_n^0|}{|U_{wn}|}, \quad a_\tau = \frac{|U_\tau^0|}{|U_{w\tau}|}$$

and making the assumption that the mass and velocity before and after impact are the same for all particles, we obtain the following expressions for the projections on the Ox and Oy axes of the force exerted by the particles on the plate:

$$X = \int_\sigma \rho_w U_{wn} \left[U_{wn} (a_n + 1) \sin\alpha - U_{w\tau}/(a_\tau - 1) \cos\alpha\right] d\sigma,$$

$$Y = \int_\sigma \rho_w U_{wn} \left[U_{wn} (a_n + 1) \cos\alpha + U_{w\tau} (a_\tau - 1) \sin\alpha\right] d\sigma. \qquad (1)$$

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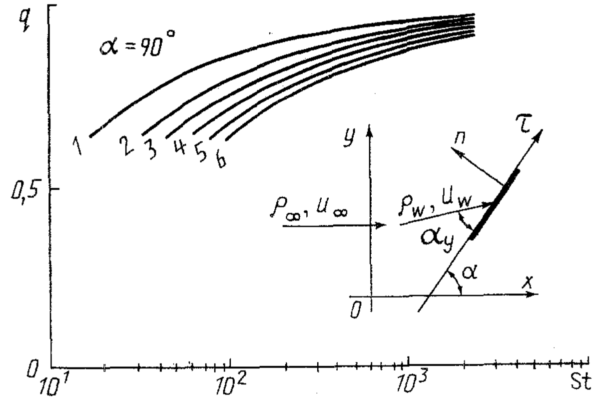

Fig. 1. The coefficient q of the velocity head of a flow of a discrete phase for a plate positioned transversely to the flow: 1) Φ = 100; 2) 550; 3) 1100; 4) 1650; 5) 2700; 6) 4100.

Under the condition that the coefficients of restitution of the particles are constant on the entire plate and also that $a_y = a$ (see Fig. 1), the system of equations (1) can be solved for $a_n$ and $a_\tau$:

$$a_n = \frac{|X| + |Y| \operatorname{ctg} \alpha}{q \rho_\infty U_\infty^2 S} - 1, \quad a_\tau = \frac{|Y| \operatorname{tg} \alpha - |X|}{q \rho_\infty U_\infty^2 S} + 1, \tag{2}$$

where

$$q = \frac{\int\limits_S \rho_w U_w^2 dS}{\rho_\infty U_\infty^2 S}.$$

The results of the experimental investigation of collisional interaction of bodies [1-3] showed that the parameters which affect the coefficients of restitution are the velocity and angle of contact of a particle with the surface, the size of the particle, and also the physical and mechanical properties of the materials of the surface and of the particle:

$$a_n, a_\tau = f(U_w, \alpha_y, d, C_T, C_s).$$

In the general case the functional dependence of the coefficients $a_n$ and $a_\tau$ on the parameters determining them is unknown. The particular case when the coefficients $a_n$ and $a_\tau$ are constant is the case for which the parameters $u_w$, $a_y$, d, $C_t$, and $C_s$ are constant and identical for each particle colliding with the surface of the plate. Thus the requirement which an experiment on measuring the coefficients $a_n$ and $a_\tau$ must meet is that the parameters $u_w$, $a_y$, d, $C_t$, and $C_s$ must be constant over the entire surface of the plate being tested.

The main factors which prevent this condition from being satisfied are interaction of the gas and the solid phases in the disturbed zone of the flow of gas suspension near the model and the formation of a "protective" layer from the particle reflected from the surface.

To estimate the degree to which the interphase interaction affects the measurements of the coefficients $a_n$ and $a_\tau$ a stationary flow of incompressible liquid, carrying solid particles near a flat wedge, was studied. It was assumed that the concentration of the solid phase in the flow is insignificant, so that the reverse effect of the solid phase on the carrying phase can be neglected. In calculating the motion of the particles only the aerodynamic drag force was taken into account. The aerodynamic drag for the particles comprising the solid phase used in the experiment was found in [4]. The velocity field of the gas near the model being tested was described using the relation of [5], which determines discontinuous flow around a symmetric wedge with unit velocity at infinity. The algorithm for calculating the flow of a gas suspension is given in [6].

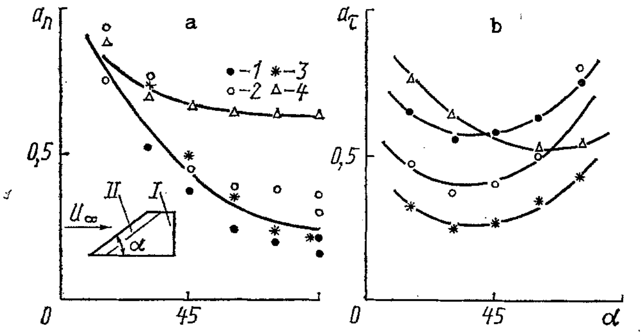

Fig. 2. The coefficient of restitution for the normal (a) and tangential (b) components of the particle velocity for different materials ($U_w$ = 84 m/sec; d = 109 μm): 1) steel; 2) copper; 3) lead; 4) glass. α, deg.

The computational results showed, under the condition that the Stokes number St > 1000 and Φ < 2000, that the angle of contact $a_y$ of the particles with the surface differs from the angle α by not more than 2-3° and the collision velocity $U_w$ is constant to within not worse than 1% on the entire surface of the wedge. The coefficient q, which shows the degree to which the velocity head of the discrete phase changes owing to stopping particles in the disturbed zone of the flow of the mixture near the body, decreases as the parameter Φ increases and as the Stokes number decreases. As the angle α increases q decreases. Figure 1 shows as an example the dependence of the coefficient q on the parameters St and F for a ribbon, placed transverse to the flow (α = 90°). The computational results made it possible to find the region in which the characteristic parameters of the flow change (St > 1000, Φ < 2000); in this region the main requirement imposed on the experiment for determining the coefficients $a_n$ and $a_\tau$, that $U_w$ and $a_y$ be constant on the entire surface of the model, is satisfied. Evidently, the use of plates, made of a uniform material as well as narrow fractions of powders in the experiment makes it possible to satisfy the condition, when d, $C_t$, and $C_s$ are constant.

To estimate the effect of the protective layer on the measurement of the coefficients $a_n$ and $a_\tau$ we shall determine the probability that an incident particle will pass through the protective layer without colliding with particles reflected from the surface. Let the particles be solid spheres. We neglect the effect of the gas phase on the parameters of the motion of the incident particles. If the effective cross section of the collisions is taken to be $\pi d^2$, then it can be shown that the free path of an incoming particle in the medium of the reflected particles is

$$\lambda = \frac{U_\infty}{n^0 \pi d^2 U_R}.$$

The relative velocity of the particles and the number density of the reflected particles in the protective layer on the surface of the body can be represented in the form $U_{Rw}$ = $U_\infty \sqrt{(1+a_n)^2 \sin^2 \alpha + (1-a_\tau)^2 \cos^2 \alpha}$ and $n_w^0 = \frac{6\rho_\infty}{\rho_p \pi d^3 a_n}$, respectively.

Examining the dependence of the efficiency of screening by the protective layer on the angle of inclination of the plate, it is obvious that the screening efficiency will be highest when the plate is positioned transverse to the flow. For this reason the error introduced by the protective layer in the measurements of the coefficients $a_n$ and $a_\tau$ was estimated for a plate positioned at an angle of α = 90°.

It is well known that the relative velocity $U_R$ and the density $n^0$ of the reflected particles do not remain constant as the distance from the surface of the body increases. It was assumed that in the protective layer at the plate positioned transverse to the flow as the distance to the surface decreases the relative velocity and density of the reflected particles increase linearly from the value of the velocity $U_\infty$ of the incident particles up to $U_{Rw}$ and from zero to $n_w^0$, respectively. The thickness L of the protective layer was taken as the greatest distance to which a particle recoils from the surface of the plate (a = 90°) along

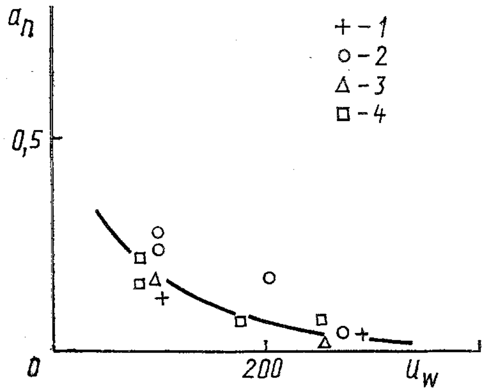

Fig. 3. Effect of incident velocity of the particles striking the plate positioned transverse to the flow on the coefficient of restitution. The plate is made of steel and a = 90°. d = 23 (1), 32 (2), 88 (3), and 109 (4) μm. $u_w$, m/sec.

the zero stream lime of the carrying phase. Then, using Clausius's formula [7] from the elementary kinetic theory of matter, it is possible to obtain an expression for the probability that the incident particle will pass through the protective layer without colliding with the particles reflected from the surfaces:

$$P = \exp\left[-\frac{6\rho_\infty L}{\rho_p d a_n}\left(\frac{a_n}{3}+\frac{1}{2}\right)\right].$$

One can see that in order to determine the probability P it is necessary to know the coefficient $a_n$. The thickness L of the protective layer is calculated from the value of $a_n$ determined experimentally. The calculations show that in order for the probability P to be less than 0.95 the experiments on determining $a_n$ and $a_\tau$ must be performed in a flow in which the density of the solid phase does not excedd 0.1 kg/m$^2$ (the average mass concentration of the solid phase in the flow $K < 0.05$).

The coefficients $a_n$ and $a_\tau$ were investigated experimentally using a setup consisting of a tube of the cylinder-ejector type [8]. The solid-phase dispenser regulates the flow rate of the component through a nozzle in the range 0.01-2.7 kg/sec. The operating regime of the dispenser is determined by the required concentration of the solid phase in the flow.

Standard electrocorundum micro- and grinding powders were used for the solid phase. The use of these powders with a quite narrow particle-size distribution function makes it possible to study the effect of the particle size of the coefficient of restitution. The powders consist of particles in the form of sharp-grained elongated splinters, the density of whose material $\rho_p$ = 4000 kg/m$^3$.

A high-velocity flow of a gas suspension has a significant erosion power. For this reason, in order to reduce the change in shape and physical properties of the material of the model the duration of the experiment must be reduced as much as possible. For this, a controlling measuring complex was constructed based on a microcomputer. The residence time of the model in the flow did not exceed 3-5 sec.

The investigations were performed on the models shown in Fig. 2a. Replaceable plates II were fastened on the casing I of the model; the purpose of the plates was to investigate the coefficients $a_n$ and $a_\tau$ for different materials. Six casings with a different vertex angle were prepared: α = 15, 30, 45, 60, 75, and 90°. The transverse section of the casing of the model is a 25 × 25 mm$^2$ square.

In order to determine, with the help of the relations (2), the coefficients $a_n$ and $a_\tau$ it is necessary to measure the force exerted on the plate by the particles, and the density and velocity of the solid phase in the incident flow. The force loads acting on the model in the flow of the gas suspension were measured with the help of aerodynamic strain balances. The force exerted on the plate under study by only the particles was determined as the difference between the resistance force of the plate in the two-phase flow and the resistance force of the same plate in the flow of pure air [9]. The velocity and density of the solid phase in the flow were measured by laser optical methods [10].

The experimental investigation of the coefficients of restitution of the particles was performed on plates made of the following materials: St3 steel, M3 copper, lead, and K8 glass. The results showed that the main parameter affecting the coefficients $a_n$ and $a_\tau$ is the angle of incidence of the particles on the surface. Figure 3a shows the dependence of the coefficient of restitution for the normal velocity component of the particles on the angle of incidence. The coefficient $a_n$ decreases monotonically as the angle a increases. This is apparently connected with the fact that as the angle a increases a larger fraction of the particle energy is expended on plastic strain and formation of defects on the surface of the plate. Among the materials studied, only glass has a higher coefficient $a_n$. This is evidently explained by the fact that the physical and mechanical properties of the glass are markedly different from those of the other materials: glass is a hard and brittle material. But the character of the dependence of the coefficient $a_n$ on the angle a is the same for all the materials studied.

The effect of the angle of inclination of the plate on the coefficient of restitution for the tangential component of the particle velocity is shown in Fig. 2b. It is evident that this dependence has a distinct extremum – at some angle a the coefficient $a_\tau$ is minimum. As the angle a increases the particle is embedded deeper into the material of the plate; this increases the resistance to the motion of the particle in the tangential direction. The coefficient $a_\tau$ decreases to angles a at which the maximum amount of material is removed as a result of erosion. As the angle a increases further more particle energy is expended on deformation of the material in the normal and not the tangential directions to the surface, so that the coefficient $a_n$ continues to decrease and the coefficient $a_\tau$ increases.

The average standard deviation of the measurements of the coefficients $a_n$ and $a_\tau$ was equal to 0.17 and 0.08, respectively.

The results of the investigations presented here permit drawing the following conclusions.

1. Questions connected with the experimental determination of the coefficients of restitution for the normal and tangential components of the particle velocity based on the force exerted by the particles on a plate positioned at an agle to the direction of flow of a gas suspension were studied. The conditions under which the measurements must be performed were established: the Stokes number $St > 1000$, the parameter $\Phi < 2000$, and the density of the discrete phase in the flow must not exceed 0.1 $kg/m^3$.

2. The coefficients of restitution of the particles were determined for steel copper, lead, and glass. The main parameters which determine the value of the coefficient of restitution are the angle of incidence of the particles and the physical and mechanical properties of the material of the surface.

## NOTATION

$a_n$ and $a_\tau$, coefficients of restitution for the normal and tangential components of the particle velocity; $C_T$ and $C_S$, parameters which characterize the physical and mechanical properties of the plate and particle materials; D, characteristic size of the body; d, average mass size of the particles; K, average mass flow concentration of the solid phase; L, thickness of the protective layer of reflected particles; n, number density of the particles; P, probability that a particle will pass through the protective layer without undergoing a collision; q, velocity head coefficient of the discrete phase; S, area of the projection of the plate on a surface perpendicular to the velocity vector of the incident flow; U, particle velocity; $U_R$, relative velocity of the incident and reflected particles; V, velocity of the gas; X and Y, projections of the force exerted by the particles on the plate along the Ox and Oy coordinate axes; a, angle of inclination of the plate to the direction of the incident flow; $a_y$, angle of incidence of the particle on the surface; λ, mean free path; μ, coefficient of dynamic viscosity of the gas; ρ, density of the discrete phase (the mass of the particles per unit volume of the mixture); $\rho_p$, density of the particle material; $\rho_G$, density of the gas; σ, area of the plate; $St=d^2\rho_p V_\infty/18\mu D$ Stokes number; $\Phi=18\rho^2_G V_\infty D/\mu\rho_p$. The indices are as follows: $\infty$, parameters referring to the undisturbed flow; w, parameters at the surface of the plate; $a^0$, parameters referring to the reflected particles; and n and $\tau$, projections normal and tangential to the surface of the plate.

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